

Abstract

The point graph of a partial geometry is the graph whose vertices are the points of the geometry and adjacency is defined by collinearity. The point graph associated to a partial geometry is a strongly regular graph and, in this case, the strongly regular graph is named *geometric*. When the parameters of a strongly regular graph, Γ , satisfy the relations of a geometric graph, then Γ is named a *pseudo geometric* graph. Not every pseudo geometric graph is geometric. In this work, we characterize strongly regular graphs that are pseudo geometric and we analyze when the complement of a pseudo geometric graph is also pseudo geometric.

Strongly Regular Graph

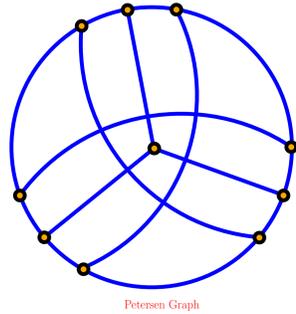
A *strongly regular graph* with parameters (n, k, λ, μ) is a graph on n vertices which is regular of degree k , any two adjacent vertices have exactly λ common neighbours and two non-adjacent vertices have exactly μ common neighbours.

If Γ is not a conference graph, then there exist $h \geq 1$ and $\beta \in \mathbb{Z}$ s.t.

$$(\mu - \lambda)^2 + 4(k - \mu) = h^2$$

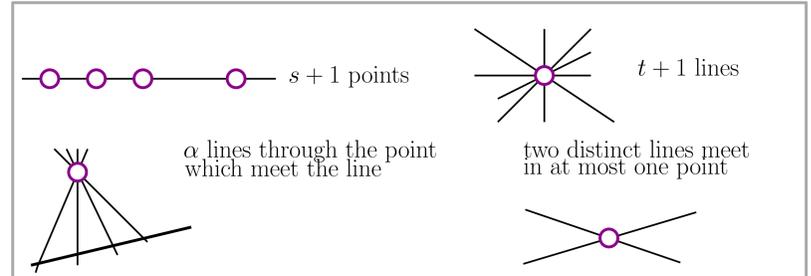
$$(n - 1)(\mu - \lambda - h) = 2(k + h\beta)$$

$$\nu = \frac{1}{2}(h + \mu - \lambda)$$

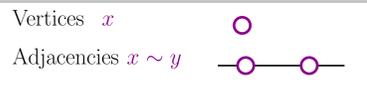


Partial geometry

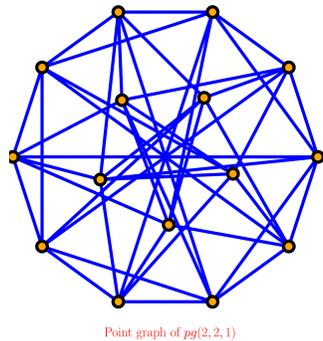
A *partial geometry* with parameters $s, t, \alpha \geq 1$, $pg(s, t, \alpha)$, is an incidence structure of points and lines such that every line has $s + 1$ points, every point is on $t + 1$ lines, two distinct lines meet in at most one point and given a line and a point not in it, there are exactly α lines through the point which meet the line.



The point graph of $pg(s, t, \alpha)$ as a strongly regular graph

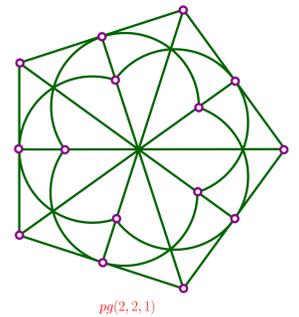


- ▶ $s(t + 1)$ -regular
- ▶ $\alpha = s + 1 \implies K_n \longleftrightarrow \text{Linear space}$
- ▶ $\alpha \leq s \implies \text{s.r.g. with parameters } (n, s(t + 1), s - 1 + t(\alpha - 1), \alpha(t + 1))$



Pseudo-geometric graphs

A strongly regular graph is a *pseudo geometric graph* if its parameters are $(n, s(t + 1), s - 1 + t(\alpha - 1), \alpha(t + 1))$ where $1 \leq \alpha \leq \min\{s, t + 1\}$ and α divides $st(s + 1)$

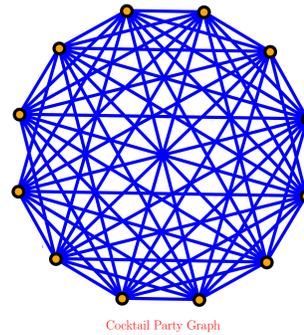


When a s.r.g. is pseudo geometric?

$\Gamma(n, k, \lambda, \mu)$ is pseudo geometric iff one of the following conditions hold:

1. Γ is a conference graph with $n = \ell^2(p^2 + q^2)^2$, where ℓ, p are odd integers, q is even and $\gcd(p, q) = 1$. Moreover, μ is even and Γ is a dual transversal design whose corresponding pseudo-latin square graph is $PL_{\frac{1+\sqrt{n}}{2}}(\sqrt{n})$.
2. Γ is not a conference graph, ν divides both μ and k and moreover $\mu \leq \nu^2$. When Γ is antipodal, this property holds iff $k \leq (n - k)^2$.

Antipodal s.r.g. are pseudo geometric



The parameters of a antipodal Γ are $((s + 1)(t + 1), s(t + 1), (s - 1)(t + 1), s(t + 1))$ Γ is bipartite iff $s = 1$ and the bipartite strongly regular graphs are $K_{2 \times (t+1)}$, $t \geq 1$, and all of them are pseudo geometric. The graph $K_{(s+1) \times 2}$, known as *Cocktail Party Graph*, is pseudo geometric iff $s \leq 2$.

The complement of a pseudo geometric graph is pseudo geometric?

If Γ is pseudo geometric, then its complement $\bar{\Gamma}$ is also a pseudo geometric graph iff α divides st and $(s - \alpha)(t - \alpha) \leq \alpha$. Its geometrical parameters are

$$\left(\frac{st}{\alpha}, s - \alpha, \frac{t(s - \alpha)}{\alpha} \right)$$

Families of s.r.g., other than antipodal, that are pseudo geometric

The Kneser graph $K(m, 2)$, $m \geq 4$, is pseudo geometric iff m is even.

The Negative-Latin square $NL_r(m)$, where $1 \leq r < m \leq r(r + 3)$ is pseudo geometric iff $r(r + 1) \leq (m - r)^2$ and $m - r$ divides $r(r + 1)$ and $r(m + 1)$

A symmetric balanced incomplete block design Γ , $(n, s(t + 1), (s - t - 1)(t + 1), (s - t - 1)(t + 1))$ iff $t + 2 \leq s \leq 2(t + 1)$.

The graphs $NL_r(2r)$, $r \geq 1$, are the unique Negative-Latin squares that are symmetric balanced incomplete block designs and they are **not** pseudo geometric.